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USN | | | | | | | | | | | **10MAT41**

Fourth Semester B.E. Degree Examination, December 2012

Engineering Mathematics – IV

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Using the Taylor's series method, solve the initial value problem $\frac{dy}{dx} = x^2y - 1$, $y(0) = 1$ at the point $x = 0.1$ **(06 Marks)**

b. Employ the fourth order Runge-Kutta method to solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$, $y(0) = 1$ at the points $x = 0.2$ and $x = 0.4$. Take $h = 0.2$. **(07 Marks)**

c. Given $\frac{dy}{dx} = xy + y^2$, $y(0) = 1$, $y(0.1) = 1.1169$, $y(0.2) = 1.2773$, $y(0.3) = 1.5049$. Find $y(0.4)$ using the Milne's predictor-corrector method. Apply the corrector formula twice. **(07 Marks)**

2 a. Employing the Picard's method, obtain the second order approximate solution of the following problem at $x = 0.2$.

$$\frac{dy}{dx} = x + yz, \quad \frac{dz}{dx} = y + zx, \quad y(0) = 1, \quad z(0) = -1.$$ **(06 Marks)**

b. Using the Runge-Kutta method, find the solution at $x = 0.1$ of the differential equation $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} - 2xy = 1$ under the conditions $y(0) = 1$, $y'(0) = 0$. Take step length $h = 0.1$. **(07 Marks)**

c. Using the Milne's method, obtain an approximate solution at the point $x = 0.4$ of the problem $\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 6y = 0$, $y(0) = 1$, $y'(0) = 0.1$. Given that $y(0.1) = 1.03995$, $y(0.2) = 1.138036$, $y(0.3) = 1.29865$, $y'(0.1) = 0.6955$, $y'(0.2) = 1.258$, $y'(0.3) = 1.873$. **(07 Marks)**

3 a. If $f(z) = u + iv$ is an analytic function, then prove that $\left(\frac{\partial}{\partial x}|f(z)|\right)^2 + \left(\frac{\partial}{\partial y}|f(z)|\right)^2 = |f'(z)|^2$. **(06 Marks)**

b. Find an analytic function whose imaginary part is $v = e^x\{(x^2 - y^2)\cos y - 2xy\sin y\}$. **(07 Marks)**

c. If $f(z) = u(r, \theta) + iv(r, \theta)$ is an analytic function, show that u and v satisfy the equation $\frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\varphi}{\partial\theta^2} = 0$. **(07 Marks)**

4 a. Find the bilinear transformation that maps the points 1, i, –1 onto the points i, 0, –i respectively. **(06 Marks)**

b. Discuss the transformation $W = e^z$. **(07 Marks)**

c. Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)}dz$, where C is the circle $|z| = 3$. **(07 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – B

5 a. Express the polynomial $2x^3 - x^2 - 3x + 2$ in terms of Legendre polynomials. **(06 Marks)**

b. Obtain the series solution of Bessel's differential equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0$ in the form $y = AJ_n(x) + BJ_{-n}(x)$. **(07 Marks)**

c. Derive Rodrique's formula $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2 - 1)^n$. **(07 Marks)**

6 a. State the axioms of probability. For any two events A and B, prove that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. **(06 Marks)**

b. A bag contains 10 white balls and 3 red balls while another bag contains 3 white balls and 5 red balls. Two balls are drawn at ransom from the first bag and put in the second bag and then a ball is drawn at random from the second bag. What is the probability that it is a white ball? **(07 Marks)**

c. In a bolt factory there are four machines A, B, C, D manufacturing respectively 20%, 15%, 25% 40% of the total production. Out of these 5%, 4%, 3% and 2% respectively are defective. A bolt is drawn at random from the production and is found to be defective. Find the probability that it was manufactured by A or D. **(07 Marks)**

7 a. The probability distribution of a finite random variable X is given by the following table:

x_i	-2	-1	0	1	2	3
$p(x_i)$	0.1	k	0.2	2k	0.3	k

Determine the value of k and find the mean, variance and standard deviation. **(06 Marks)**

b. The probability that a pen manufactured by a company will be defective is 0.1. If 12 such pens are selected, find the probability that (i) exactly 2 will be defective, (ii) at least 2 will be defective, (iii) none will be defective. **(07 Marks)**

c. In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation, given that A(0.5) = 0.19 and A(1.4) = 0.42, where A(z) is the area under the standard normal curve from 0 to z >0. **(07 Marks)**

8 a. A biased coin is tossed 500 times and head turns up 120 times. Find the 95% confidence limits for the proportion of heads turning up in infinitely many tosses. (Given that $z_c = 1.96$) **(06 Marks)**

b. A certain stimulus administered to each of 12 patients resulted in the following change in blood pressure:

5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4 (in appropriate unit)

Can it be concluded that, on the whole, the stimulus will change the blood pressure. Use $t_{0.05}(11) = 2.201$. **(07 Marks)**

c. A die is thrown 60 times and the frequency distribution for the number appearing on the face x is given by the following table:

x	1	2	3	4	5	6
Frequency	15	6	4	7	11	17

Test the hypothesis that the die is unbiased.
(Given that $\chi^2_{0.05}(5) = 11.07$ and $\chi^2_{0.01}(5) = 15.09$) **(07 Marks)**

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10CS42

Fourth Semester B.E. Degree Examination, December 2012
Graph Theory and Combinatorics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions selecting at least two questions from each part.***

PART – A

1 a. Define connected graph. Prove that a connected graph with n vertices has at least (n – 1) edges. **(06 Marks)**

b. Define isomorphism of two graphs. Determine whether the two graphs (Fig.Q.1(b)(i)) and (Fig.Q.1(b)(ii)) are isomorphic. **(07 Marks)**

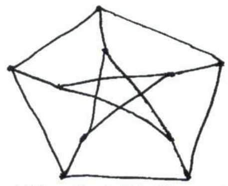


Fig.Q.1(b)(i)

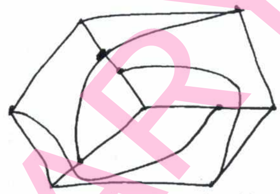


Fig.Q.1(b)(ii)

c. Define a complete graph. In the complete graph with n vertices, where n is an odd number ≥ 3, show that there are $\frac{(n-1)}{2}$ edge disjoint Hamilton cycles. **(07 Marks)**

2 a. Design a regular graph with an example. Show that the Peterson graph is a non planar graph. **(07 Marks)**

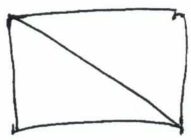
b. Prove that a graph is 2-chromatic if and only if it is a null bipartite graph. **(06 Marks)**

c. Define Hamiltonian and Eulerian graphs. Prove the complete graph $K_{3,3}$ is Hamiltonian but not Eulerian. **(07 Marks)**

3 a. Define a tree. Prove that a connected graph is a tree if it is minimally connected. **(06 Marks)**

b. Define a spanning tree. Find all the spanning trees of the graph given below. (Fig.Q.3(b)). **(07 Marks)**

Fig.Q.3(b)



c. Construct a optimal prefix code for the symbols a, o, q, u, y, z that occur with frequencies 20, 28, 4, 17, 12, 7 respectively. **(07 Marks)**

4 a. Define matching edge connectivity and vertex connectivity. Give one example for each. **(06 Marks)**

b. Using Prim's algorithm, find a minimal spanning tree for the weighted graph shown in the following Fig.Q.4(b). **(07 Marks)**

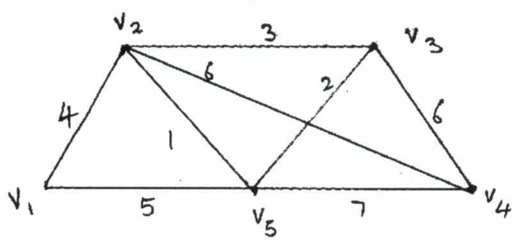


Fig.Q.4(b)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
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c. Three boys b_1, b_2, b_3 and four girls g_1, g_2, g_3, g_4 are such that
b_1 is a cousin of g_1, g_2 and g_4
b_2 is a cousin of g_2 and g_4
b_3 is a cousin of g_2 and g_3.
If a boy must marry a cousin girl, find possible sets of such couples. **(07 Marks)**

PART – B

5 a. Find the number of ways of giving 10 identical gift boxes to six persons A, B, C, D, E, F in such a way that the total number of boxes given to A and B together does not exceed 4. **(06 Marks)**

b. Define Catalan numbers. In how many ways can one travel in the xy plane from (0, 0) to (3, 3) using the moves R: (x + 1, y) and U: (x, y + 1) if the path taken may touch but never rise above the line y = x? Draw two such paths in the xy plane. **(07 Marks)**

c. Determine the coefficient of
i) xyz^2 in the expansion of $(2x - y - z)^4$
ii) $a^2b^3c^2d^5$ in the expansion of $(a + 2b - 3c + 2d + 5)^{16}$. **(07 Marks)**

6 a. How many integers between 1 and 300 (inclusive) are
i) divisible by 5, 6, 8?
ii) divisible by none of 5, 6, 8? **(07 Marks)**

b. In how many ways can the integers 1, 2, 3…..10 be arranged in a line so that no even integer is in it natural place? **(06 Marks)**

c. Find the rook polynomial for the following board (Fig.Q.6(c)). **(07 Marks)**

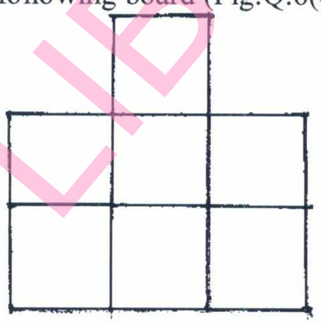

Fig.Q.6(c)

7 a. Find the coefficient of x^{18} in the following products:
i) $(x + x^2 + x^3 + x^4 + x^5)(x^2 + x^3 + x^4 + x^5 + \ldots)^5$
ii) $(x + x^3 + x^5 + x^7 + x^9)(x^3 + 2x^4 + 3x^5 + \ldots)^3$. **(07 Marks)**

b. Using the generating function find the number of i) non negative and ii) positive integer solutions of the equation $x_1 + x_2 + x_3 + x_4 = 25$. **(06 Marks)**

c. Find all the partitions of x^7. **(07 Marks)**

8 a. Solve the Fibonacci relation
$F_{n+2} = F_{n+1} + F_n$ for $n \geq 0$ given $F_0 = 0$, $F_1 = 1$. **(07 Marks)**

b. Solve the recurrence relation
$a_{n-2}\ a_{n-1} + a_{n-2} = 5_n$. **(07 Marks)**

c. Find a generating function for the recurrence relation
$a_r + 5a_{r-1} + 6a_{r-2} = 3r^2$, $r \geq 2$. **(06 Marks)**

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Fourth Semester B.E. Degree Examination, December 2012
Design and Analysis of Algorithm

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting atleast TWO questions from each part.***

PART – A

1 a. Define asymptotic notations. **(03 Marks)**

b. Algorithm X(int N)

```
{
  int P ;
  for i ← 1 to N
    {
      printf ("\n % d\ t * \t % d = %d", N, i, P);
        P = P + N ;
    } }
```

i) What does this algorithm compute?

ii) What is the basic operation?

iii) How many times the basic operation is executed?

iv) What is the efficiency class of this algorithm? **(04 Marks)**

c. Solve the following recurrence relations.

$$f(n) = \begin{cases} f(n-1)+n & n>0 \\ 0 & n=0 \end{cases}$$

$x(n) = 3x(n-1)$ for $n > 1$, $x(1) = 4$

$x(n) = x(n\,|\,2)+n$ for $n > 1$, $x(1) = 1$ $n = 2^k$. **(08 Marks)**

d. Sort the list E X A M P L E by bubble sort, Is there any possibility that bubble sort can be stopped earlier? **(05 Marks)**

2 a. Discuss how quick sort works to sort an array. Trace quick sort algorithm for the following data set 65, 70, 75, 80, 85, 60, 55, 50, 45. Also derive the worst case complexity of quick sort. **(09 Marks)**

b. Write the recursive algorithm for merge sort. **(04 Marks)**

c. Consider the following set of 14 elements in an array list, -15, -6, 0, 7, 9, 23, 54, 82, 101, 112, 125, 131, 142, 151 when binary search is applied on these elements, find the elements which required maximum number of comparisons. Also determine average number of key comparisons for successful search and unsuccessful search. **(04 Marks)**

d. Derive the time complexity for defective chess board. **(03 Marks**

3 a. Solve the following instance of knapsack problem, using greedy algorithm

Item	1	2	3	4
Weight	4	7	5	3
Profit	40	42	25	12

Knapsack weight M = 10. **(05 Marks)**

b. Using Prim's algorithm, determine minimum cost spanning tree for the following graph

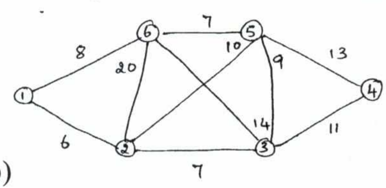

Fig. Q3(b)

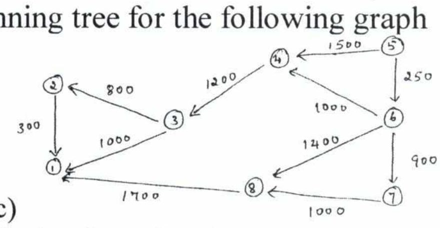

Fig. Q3(c)

How Knapsack and Prim's algorithms guarantee the elimination of cycles? **(07 Marks)**

c. In the above graph Fig. Q3(C), determine the shortest distances from source vertex 5 to all the remaining vertices, using Dijikstra's algorithm. **(08 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

4 a. Solve the following traveling sales person problem, using dynamic programming **(10 Marks)**

$$\begin{bmatrix} 0 & 10 & 15 & 20 \\ 5 & 0 & 9 & 10 \\ 6 & 13 & 0 & 12 \\ 8 & 8 & 9 & 0 \end{bmatrix} \text{ starting city } 1$$

b. Write Warshall- Floyd algorithm. **(03 Marks)**

c. Generate the transitive closure of the graph given below. **(07 Marks)**

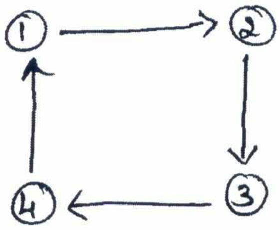

Fig. Q4(c)

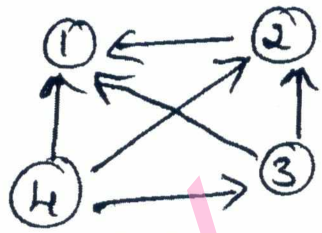

Fig. Q5(c)

PART – B

5 a. Match the pattern BAOBAB in the text BESS – KNEW – ABOUT – BAOBAS, using
   i) Horspool's algorithm
   ii) Boyer Moore algorithm. **(08 Marks)**

b. Write a BFS algorithm to check the connectivity of a given graph. **(05 Marks)**

c. Apply source elimination based algorithm to represent vertices in topological ordering for the digraph given in Fig. Q5(c). **(04 Marks)**

d. Apply distribution counting algorithm to sort the elements b, c, ,d c, b, a, a, b. **(03 Marks)**

6 a. What are decision trees? Explain with example, how decision trees are used in sorting algorithms. **(10 Marks)**

b. Explain the concepts of P, NP, and NP – complete problems. **(10 Marks)**

7 a. Draw the state – space tree to generate solutions to 4 – Queen's problem. **(04 Marks)**

b. Apply backtracking method to solve subset sum problem for the instance n = 6, d = 30.
S = {5, 10, 12, 13, 15, 18} **(06 Marks)**

c. What is branch – and - bound algorithm? How it is different from backtracking? **(05 Marks)**

d. Write the steps and apply nearest neighbour approximation algorithm on the TSP problem with the starting vertex a, and calculate the accuracy ratio of approximation. **(05 Marks)**

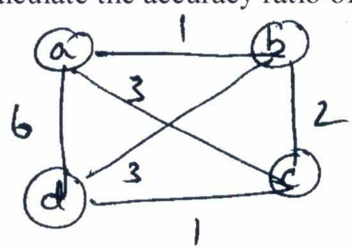

Fig. 7(d)

8 a. What are the different computation models? Discuss in detail. **(10 Marks)**

b. Let the input to the prefix computation problem be 5, 12, 8, 6, 3, 9, 11, 12, 5, 6, 7, 10, 4, 3, 5 and let ⊕ stand for addition. Solve the problem using work optimal algorithm. **(10 Marks)**

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10CS44

Fourth Semester B.E. Degree Examination, December 2012

UNIX and Shell Programming

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Explain salient features of UNIX operating system. **(07 Marks)**

b. Compare internal and external commands in UNIX with suitable example. Explain why cd command cannot be an external command. **(06 Marks)**

c. Illustrate with a diagram typical UNIX file system and explain different types of files supported in UNIX. **(07 Marks)**

2 a. Explain the basic file attributes displayed by ℓs – ℓ command. **(06 Marks)**

b. Discuss relative and absolute methods for changing file permissions. **(06 Marks)**

c. Explain with a diagram the different modes of V_i editor and list the commands in each mode. **(08 Marks)**

3 a. Explain with an example use of single quote, double quote and back quote in a command line. **(06 Marks)**

b. Explain the following commands:

i) cp ????? progs ii) kill –S KILL 121 122

iii) wc –ℓ < user.txt iv) ps – e ℓ **(06 Marks)**

c. Explain the mechanism of process creation using system calls in UNIX. **(04 Marks)**

d. Explain the following environment variables:

i) PATH ii) HISTSIZE iii) PS2 iv) SHELL **(04 Marks)**

4 a. Discuss with example hard link and soft link applicable to UNIX files. **(06 Marks)**

b. Explain the following commands:

i) umask 022

ii) find / ! – name "*.C" – Print

iii) tr –d ':/' < emp.txt

iv) touch – m 0303 10 30 vtu.txt **(08 Marks)**

c. Explain the following filters with options:

i) Paginate – Pr

ii) Sort – Sort **(06 Marks)**

PART – B

5 a. Explain with example basic regular expressions. **(06 Marks)**

b. Locate lines longer than 100 and smaller than 150 characters using (i) grep, (ii) sed. **(04 Marks)**

c. Discuss stream editor – sed with options. **(06 Marks)**

d. How do these expressions differ:

i) [0 – 9] * and [0 – 9] [0 – 9] *

ii) ^ [^ ^] and ^ ^ ^ **(04 Marks)**

6 a. What is shell programming? Write a shell program to create a menu and execute a given option based on users choice. Options include (i) list of users, (ii) list of processes, (iii) list of files. **(06 Marks)**

b. Explain with example set and shift commands in UNIX to manipulate positional parameters. **(04 Marks)**

c. Discuss use of trap statement for interrupting a program in UNIX. **(04 Marks)**

d. Explain with an example while and for loop in shell programming. **(06 Marks)**

7 a. Write a note on awk and explain built in variables in awk. **(08 Marks)**

b. Explain with example the following awk function:
i) Split ( )   ii) Substr ( )   iii) length ( )   iv) index ( ) **(08 Marks)**

c. i) Write an awk statement to print odd numbered lines in a file.
ii) Write an awk statement to delete blank lines from a file. **(04 Marks)**

8 a. Explain string handling function in perl. **(06 Marks)**

b. Using command line arguments, write a perl program to find whether a given year is a leap year. **(07 Marks)**

c. Write a perl program to convert a given decimal number to binary equivalent. **(07 Marks)**

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Fourth Semester B.E. Degree Examination, December 2012
Computer Organization

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting atleast TWO questions from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Explain the different functional units of a digital computer. **(05 Marks)**

b. Draw and explain the connection between memory and processor with the respective registers. **(05 Marks)**

c. Explain clearly SPEC rating and its significance. Assuming that the reference computer is ultra SPARCIO work station with 300 MHz ultra SPARC processor. A company has to purchase 1000 new computers hence ordered testing of new computer with SPEC 2000. Following observation were made.

Programs	Runtime on reference computer	Runtime in new computer
1	50 minutes	5 Minutes
2	75 Minutes	4 Minutes
3	60 Minutes	6 Minutes
4	30 Minutes	3 Minutes

The company system manger will place the order for purchasing new computers only if the overall SPEC rating is atleast 12. After the said test will the system manger place order for purchase of new computer. **(10 Marks)**

2 a. What is little endian and big endian memory? Represent the number 64243848H in 32 bits big endian and little endian memory. **(06 Marks)**

b. What is addressing mode? Explain immediate, direct and indirect addressing mode by an example. **(06 Marks)**

c. Explain logical shift and rotate instructions, with examples. **(08 Marks)**

3 a. Define memory mapped I/O and IO mapped I/O, with examples. **(05 Marks)**

b. Explain how interrupt requests from several IO devices can be communicated to a processor through a single INTR line. **(10 Marks)**

c. What are the different methods of DMA? Explain them in brief. **(05 Marks)**

4 a. With a block diagram, explain how the keyboard is connected to processor. **(06 Marks)**

b. Explain the serial port and serial interface. **(06 Marks)**

c. Explain architecture and protocols, with respect to USB. **(08 Marks)**

PART – B

5 a. Draw a diagram and explain the working of 16 Mega bits DRAM chip configured as $2M \times 8$. Also explain as at how it can be made to work in fast page mode. **(10 Marks)**

b. Briefly explain any four non-voltile memory concepts. **(05 Marks)**

c. With figure analyse the memory hierarchy interms of speed cost and size. **(05 Marks)**

6 a. Explain the design of a four bits carry – look ahead adder circuit. **(10 Marks)**
b. Gives Booth's algorithm to multiply two binary numbers. Explain the working of algorithm by taking an example. **(10 Marks)**

7 a. Write and explain the control sequence for execution of an unconditional branch instruction. **(10 Marks)**
b. Draw and explain multiple bus organization. Explain its advantages. **(10 Marks)**

8 a. Write short note on power wall **(06 Marks)**
b. What you mean by shared memory multiprocessors. **(06 Marks)**
c. Explain the different approaches used in multithreading. **(08 Marks)**

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Fourth Semester B.E. Degree Examination, December 2012

Advanced Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Prove that the angle between two lines whose direction cosines are (ℓ_1, m_1, n_1) and (ℓ_2, m_2, n_2) is $\cos\theta = \ell_1\ell_2 + m_1m_2 + n_1n_2$. **(06 Marks)**

b. Find the projection of the line AB on CD where A = (1, 3, 5), B = (6, 4, 3), C = (2, -1, 4) and D = (0, 1, 5). **(07 Marks)**

c. Find the angle between any two diagonals of cube. **(07 Marks)**

2 a. Find the equation of the plane passing through the points (3, 1, 2) and (3, 4, 4) and perpendicular to $5x + y + 4z = 0$. **(06 Marks)**

b. Show that the points (0, -1, 0), (2, 1, -1), (1, 1, 1) and (3, 3, 0) are coplanar. **(07 Marks)**

c. Find the equation of the plane through the points (1, 0, -1), (3, 2, 2) and parallel to the line $\frac{x-1}{1} = \frac{1-y}{2} = \frac{z-2}{3}$. **(07 Marks)**

3 a. Find the value of λ such that the vectors $\lambda i + j + 2k$, $2i - 3j + 4k$ and $i + 2j - k$ are coplanar. **(06 Marks)**

b. If $\vec{a} = 4i + 2j - k$, $\vec{b} = 2i - j$ and $\vec{c} = j - 3k$, find (i) $(\vec{a}\times\vec{b}).(\vec{b}\times\vec{c})$, (ii) $(\vec{a}\times\vec{b})\times(\vec{b}\times\vec{c})$. **(07 Marks)**

c. Find the cosine and sine of the angle between the vectors $2i - j + 3k$ and $i - 2j + 2k$. **(07 Marks)**

4 a. Find the components of velocity and acceleration at t = 2 on the curve, $\vec{r} = (t^2+1)i + (4t-3)j + (2t^2 - 6t)k$ in the direction of $i + 2j + 2k$. **(06 Marks)**

b. Find the angle between the tangents to the curve $\vec{r} = \left\{t - \frac{t^3}{3}\right\}i + t^2 j + \left\{t + \frac{t^3}{3}\right\}k$ at $t = \pm 3$. **(07 Marks)**

c. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at (1, -2, -1) along $2i - j - 2k$. **(07 Marks)**

5 a. If $\vec{F} = \nabla(xy^3z^2)$, find $\operatorname{div}\vec{F}$ and $\operatorname{curl}\vec{F}$ at the point (1, -1, 1). **(06 Marks)**

b. Show that $\vec{F} = (y+z)i + (z+x)j + (x+y)k$ is irrotational. Also find a scalar function ϕ such that $\vec{F} = \nabla\phi$. **(07 Marks)**

c. Prove that $\nabla^2(\log r) = \frac{1}{r^2}$ where $\vec{r} = xi + yj + zk$ and $r = |\vec{r}|$. **(07 Marks)**

6 a. Find Laplace transform of $(2t + 3)^2$. **(05 Marks)**

b. Find Laplace transform of $e^{2t}\cos 3t$. **(05 Marks)**

c. Find $L\left\{\dfrac{\cos 2t - \cos 3t}{t}\right\}$. **(05 Marks)**

d. Using Laplace transform, evaluate $\int_0^\infty e^{-2t}\, t\cos t\, dt$. **(05 Marks)**

7 a. Find inverse Laplace transform of $\dfrac{s}{s^2 + 4s + 13}$. **(06 Marks)**

b. Find $L^{-1}\left\{\dfrac{1}{(s^2 + 3s + 2)(s + 3)}\right\}$. **(07 Marks)**

c. Find $L^{-1}\left\{\log\left(\dfrac{s^2 + 1}{s^2 + s}\right)\right\}$. **(07 Marks)**

8 a. Solve the differential equation $y'' + 4y' + 3y = e^{-t}$ with $y(0) = 1$ and $y'(0) = 1$ by using Laplace transforms. **(10 Marks)**

b. Solve by using Laplace transforms $\dfrac{dx}{dt} - 2y = \cos 2t$, $\dfrac{dy}{dt} + 2x = \sin 2t$ with $x = 1$, $y = 0$ at $t = 0$. **(10 Marks)**

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